Deflection Control During Dynamic Deformations of a Rectangular Plate Using Piezoceramic Elements

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Nomenclature

- D = induction vector
- c = material elasticities
- e = piezoelectric constants
- K = stiffness matrix
- M = generalized mass matrix
- u = displacement vector
- V = applied voltage
- δ_{ij} = Kronecker delta
- ϵ = infinitesimal strain tensor
- ξ = dielectric permittivity
- ρ = mass density
- σ = stress tensor

Introduction

THERE have been several investigations (e.g., see Baz and Poh,[1] Tzou and Tseng,[2] and Crawley and de Luis[3]) illustrating the use of piezoceramic (PZT) elements to control the vibrations of a beam. Here we illucidate the use of PZTs as sensors and actuators to control the deflection of the centroid of a rectangular plate suddenly subjected to a uniformly distributed load.

Formulation of the Problem

We consider a 0.3 m × 0.3 m × 1.8 mm aluminum plate simply supported on two opposite edges with PZTs affixed to its top and bottom surfaces as shown in Fig. 1. The technique developed herein is valid for plates of other dimensions, materials, and support conditions. The PZT element at the centroid of the plate is used as a sensor and the other PZT elements are used as actuators. We employ the first-order shear deformation theory to study elastic deformations of the plate and the PZTs. The PZTs are assumed to be perfectly bonded to the plate surfaces through adhesives of negligible thicknesses, and displacements and surface tractions across the interfaces between the PZTs and the plate are taken to be continuous. Whereas the plate material is modeled by Hooke's law and is taken to be isotropic, the PZTs are transversely isotropic with electroelastic response given by

$$\{\sigma\} = [c]\{\epsilon\} - [e]^T\{E\} \tag{1}$$

$$\{D\} = [e]\{\epsilon\} + [\xi]\{E\} \tag{2}$$

Here $E = -\text{grad}\, V$ is the electric field vector. For a transversely isotropic material, matrices c, e, and ξ are sparsely populated, e.g., see Halpin.[4] As is the practice in thin-plate theory we set $\sigma_{33} = 0$ and $\epsilon_{33} = 0$ (see Fig. 1 for the choice of coordinate axes). The displacement u of any point of the plate is taken as

$$u_i(x_1, x_2, x_3, t) = u_i^0(x_1, x_2, t) + (1 - \delta_{i3})x_3\phi_i(x_1, x_2, t) \qquad i = 1, 2, 3 \tag{3}$$

where u^0 gives the displacement of a point on the midsurface $x_3 = 0$ of the plate; ϕ_1, ϕ_2 are angles of rotation of the normal to the midsurface of the plate about the x_2 and x_1 axes, respectively, and $\phi_3 \equiv 0$.

Instead of working with the deflection equation for a plate, we start with the balance of linear momentum and the Maxwell equation

$$\rho\ddot{u}_i = \sigma_{ij,j} \tag{3}$$

$$D_{i,i} = 0 \tag{4}$$

where an overdot indicates the material time derivative, a comma followed by index j implies partial differentiation with respect to x_j, and a repeated index implies summation over the range of the index. The pertinent initial and boundary conditions are

$$u^0(x, 0) = 0, \quad \dot{u}^0(x, 0) = 0, \quad \phi(x, 0) = 0, \quad \dot{\phi}(x, 0) = 0 \tag{5}$$

$$u_2^0(x_1, x_2, t) = 0, \quad u_3^0(x_1, x_2, t) = 0, \quad \phi_1(x_1, x_2, t) = 0 \tag{6}$$

at $x_2 = 0, 0.3$ m,

$$\sigma_{i2}(x_1, x_2, x_3, t) = 0 \tag{7}$$

at $x_1 = 0, 0.3$ m.

Following Hughes[5] we derive a weak formulation of Eqs. (3) and (4). The semidiscrete formulation of Eq. (3) is

$$M\ddot{U} + KU = F + F_c \tag{8}$$

where U is the matrix of extended displacements defined at points on the midsurface of the plate, F is the resultant force corresponding to the applied loads, and F_c is the force exerted by the PZTs onto the plate or that experienced by the PZT sensor. We note that for the PZT, F_c is proportional to the applied voltage. Since a uniform voltage is to be applied to the PZTs, Eq. (4) is satisfied almost everywhere in the region occupied by the actuators. For the PZT sensor, we observe the average response and find the resultant voltage from F_c in Eq. (8). Most structural members exhibit internal damping; an approximate way to account for it is to modify Eq. (8) to

$$M\ddot{U} + C\dot{U} + KU = F + F_c \tag{9}$$

where C is the damping matrix and is generally taken to be a linear combination of M and K. Here we take

$$C = 10^{-4}(M + K) \tag{10}$$

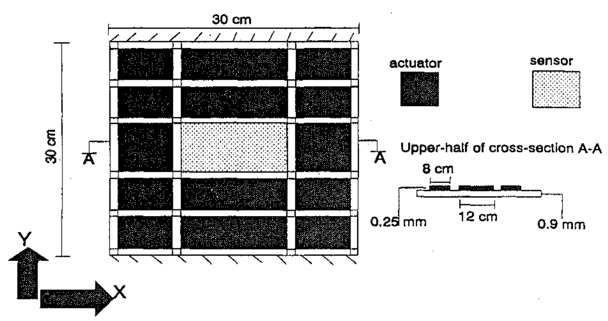

Fig. 1 Locations and sizes of PZT sensors and actuators in a "smart" plate.

Received March 13, 1995; revision received April 24, 1995; accepted for publication April 27, 1995.

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Results and Discussion

To analyze the problem just formulated, we have developed a finite element code that employs four-noded Lagrangian elements and a consistent mass matrix. All nonzero elements of the stiffness matrix except those corresponding to shearing are evaluated by using the 2×2 Gaussian quadrature rule, and those corresponding to shearing are evaluated by using the one-point quadrature rule and a shear correction factor of 5/6. The coupled linear ordinary differential equations (9) are integrated numerically by using the unconditionally stable Newmark method (e.g., see Hughes[5]) with $\delta = 0.25$ and $\gamma = 0.5$. The validation of the code is discussed in Ghosh.[6]

The results presented are based on the following values of material parameters. For aluminum, Young's modulus $E = 65$ GPa, Poisson's ratio $\nu = 0.3$, mass density $\rho = 2700$ kg/m^3, and for the G1195 PZT, $E_{11} = E_{22} = E_{33} = 63$ GPa, $\nu_{12} = \nu_{23} = \nu_{13} = 0.3$, $G_{12} = G_{23} = G_{13} = 24.2$ GPa, $\rho = 7600$ kg/m^3, $d_{31} = 16.6$ pm/V, and $\xi_{11} = \xi_{22} = \xi_{33} = 15.2$ nF/m.

For a uniformly distributed load of 10 N/m^2 suddenly applied to the upper surface of the plate at time $t = 0$ and then kept fixed, Fig. 2 shows the time history of the sensor output. The amplitude of the output steadily decreases because of the Rayleigh damping considered; some damping is also introduced by the numerical algorithm. Results plotted in Fig. 3 elucidate that actuators are effective in controlling the vertical deflection of the centroid of the plate when a displacement type controller with a gain of 5×10^{24} V/C-mm is employed. A challenging task is to design multi-input multi-output controller to simultaneously annul the deflections of several points in the plate.

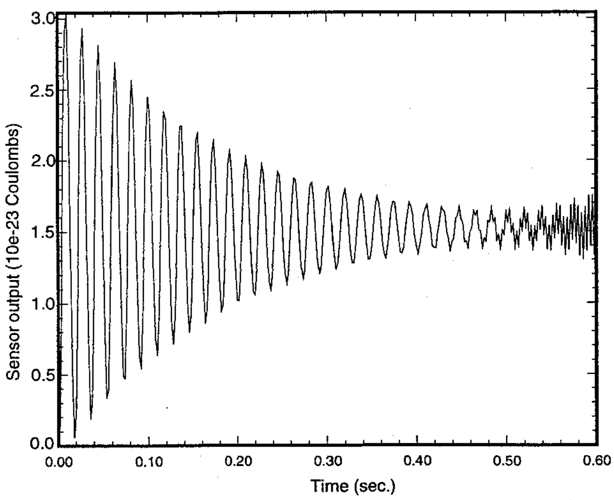

Fig. 2 Time history of the sensor output for the "smart" plate depicted in Fig. 1.

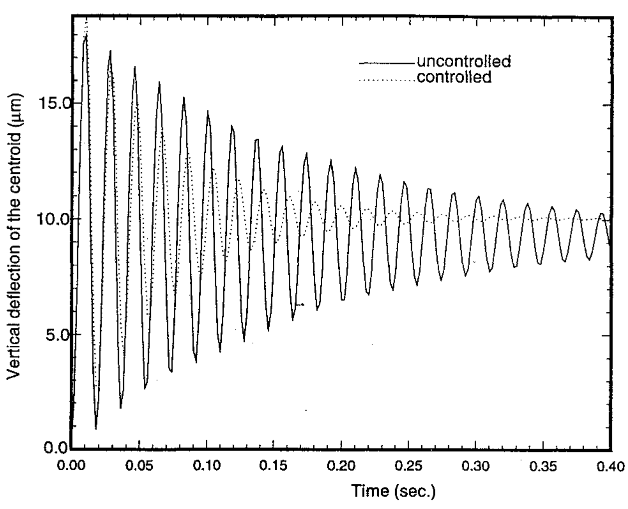

Fig. 3 Time histories of the vertical deflection of the centroid of the plate both with and without activating the actuators.

Acknowledgments

This work was supported by the U.S. Army Research Office Grant DAAH04-93-G-0214 to the University of Missouri–Rolla and a matching grant from the Missouri Research and Training Center. The Virginia Polytechnic Institute and State University acted as a subcontractor.

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Effective Transverse Young's Modulus of Composites with Viscoelastic Interphase

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Introduction

A NUMBER of studies[1–3] show that the interphase plays an essential role in the performance of fiber-reinforced composites. Papanicolaou et al.[1] reported that the interphase material is viscoelastic and evaluated its mechanical properties. Hashin[2] showed that an imperfect interface affects transverse mechanical properties of composites significantly but not axial Young's modulus. Gosz et al.[3] studied the effect of a viscoelastic interphase on the transverse properties of hexagonal array composites using two Maxwell elements and an elastic matrix.

Previous studies[2,3] carry the assumption of infinitesimally thin interfacial zones. However, the thickness of the interphase may have important consequences in material property characterization and stress distributions. A physical region may exist where material properties are altered due to chemical reaction of the adhesion process between fiber and matrix.

In the present study, time-dependent effects of interfacial layers and matrices on the mechanical properties of laminated composites with various fiber-matrix volume fractions are investigated. Both elastic and viscoelastic constitutive relations are considered for the

Received Dec. 31, 1993; revision received June 13, 1994; accepted for publication Nov. 3, 1994.

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