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THE INITIATION AND GROWTH OF ADIABATIC SHEAR BANDS

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Abstract – A simple version of thermo/viscoplasticity theory is used to model the formation of adiabatic shear bands in high rate deformation of solids. The one dimensional shearing deformation of a finite slab is considered. For the constitutive assumptions made in this paper, homogeneous shearing produces a stress/strain response curve that always has a maximum when strain and rate hardening, plastic heating, and thermal softening are taken into account. Shear bands form if a perturbation is added to the homogeneous fields just before peak stress is obtained with these new fields being used as initial conditions. The resulting initial/boundary value problem is solved by the finite element method for one set of material parameters. The shear band grows slowly at first, then accelerates sharply, until finally the plastic strain rate in the center reaches a maximum, followed by a slow decline. Stress drops rapidly throughout the slab, and the central temperature increases rapidly as the peak in strain rate develops.

I. INTRODUCTION AND FORMULATION OF THE PROBLEM

Adiabatic shear is the name given to a localization phenomena that is important in many problems involving high rate deformation of solids. In the last five years or so there has been strong interest in the theoretical aspects of the subject. CLIFTON *et al.* [1984] have listed and briefly described more than a dozen papers on rapid shearing deformation in the recent literature, as well as several of the earlier pioneering works. However, there still appears to be a need to define a theoretically complete framework for the phenomenon and to find and examine dynamic solutions within such a framework. This paper summarizes our work to date in attempting to fill that need.

A general theory of thermoplasticity, due to GREEN & NAGHDI [1965], has been taken as the starting point. According to this theory, which is rate independent, plastic strain and work hardening are modeled as internal variables controlled by evolutionary equations. In this paper those general features are retained, but in addition the yield function is taken to depend on plastic strain rates, as well as stress and temperature, in a manner similar to that used by RUBIN [1982] and DRYSDALE [1984]. This combination is completely self consistent in a thermodynamic sense and allows for smooth and continuous transitions between elastic and viscoplastic states. Details are given elsewhere, WRIGHT & BATRA [1985], and only summaries of the equations are given here.

Figure 1 shows a block of material lying between $Y = -H$ and $Y = +H$ and undergoing only horizontal motion in the X direction. This motion is volume preserving and may be written as

$$x = X + u(Y,t) \ ,$$
$$y = Y \ , \tag{1}$$
$$z = Z \ .$$

The balance relations for momentum, energy, and entropy in the absence of body forces and external sources of heat may be written

$$s,_Y = \rho \ddot{u} \ ,$$
$$\rho \dot{U} = s\dot{u},_Y - q,_Y \ , \tag{2}$$
$$\rho T \dot{\eta} - \frac{q}{T} T,_Y + q,_Y \geq 0 \ .$$

In these equations, s is the shear stress on the planes of constant Y, U is internal energy, q is heat flux due to conduction, T is temperature, η is specific entropy, and ρ is density, which is constant. The dot and the comma indicate differentiation with respect to time t and the material coordinate Y respectively, and it is assumed in the usual way that shear strain may be decomposed into elastic and plastic parts

$$\gamma = u,_Y = \gamma_e + \gamma_p \ . \tag{3}$$

With κ taken to be a measure of work hardening, it is assumed that a yield or loading function f exists such that

$$f(s, T, \dot{\gamma}_p) = \kappa \ , \tag{4}$$

where f is monotonically decreasing in $\dot{\gamma}_p$, and the criterion for elastic or plastic loading is simply

$$f(s, T, 0) \leq \kappa, \text{ elastic} \ ,$$
$$f(s, T, 0) > \kappa, \text{ plastic} \ . \tag{5}$$

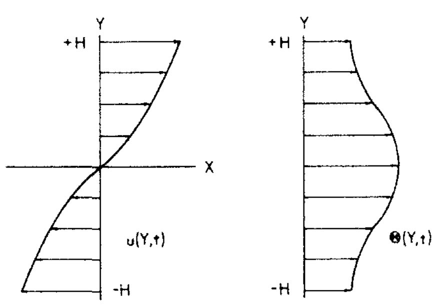

Fig. 1. Shearing of a finite block of material with displacement u and temperature change θ.

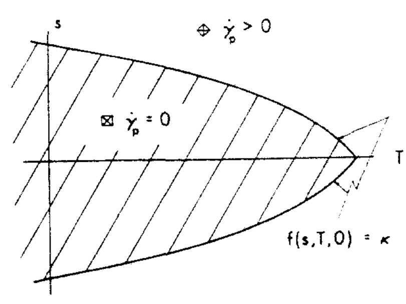

Fig. 2. Yield surface in stress-temperature space.

If plastic deformation is occurring, the sign of $\dot{\gamma}_p$ is taken to be the same as that of s, and its absolute value may be found uniquely from eqn (4) because of the assumed monotonicity of f. If plastic deformation is not occurring, then of course $\dot{\gamma}_p = 0$. The situation is shown schematically in Fig. 2. When the stress and temperature lie in the cross hatched region, deformation is wholly elastic; when they lie outside, the plastic strain rate is nonzero. Furthermore, the farther outside that the point (s, T) lies, the larger the absolute value of $\dot{\gamma}_p$. Equation (4) is similar to the yield functions used by RUBIN [1982] and DRYSDALE [1984], but the treatment that follows here is somewhat different from either of theirs. The work hardening parameter is assumed to obey the following evolutionary equation,

$$\dot{\kappa} = M\dot{\gamma}_p \ . \tag{6}$$

M is a constitutive function that depends only on s, T and κ.†

II. CONSTITUTIVE FUNCTIONS

For computational purposes, specific constitutive functions have been chosen as follows.

$$\begin{aligned} \rho U &= \frac{1}{2}\mu\gamma_e^2 + \rho T_0 c_v (e^{(\eta-\eta_0)/c_v} - 1) \ , \\ q &= -kT_{,y} \ , \end{aligned} \tag{7}$$

where μ is a constant shear modulus, T_0 is a reference temperature, c_v is the specific heat at constant volume, and k is the thermal conductivity. Standard thermodynamic arguments show that

$$s = \mu\gamma_e, \ \ T = T_0 e^{(\eta-\eta_0)/c_v} \ , \tag{8}$$

so that the elastic response is linear, and there is no thermoelastic effect.

It is further assumed for a slow isothermal reference test at temperature T_0 that $s = \kappa = \hat{\kappa}(\epsilon)$, where ϵ is the plastic strain in that case, and that whatever the rate of deformation, κ depends only on the plastic work done. Thus it follows that

$$\begin{aligned} \dot{W}_p &= \kappa\dot{\epsilon} = s\dot{\gamma}_p \ , \\ M &= \frac{1}{\kappa}\frac{d\hat{\kappa}}{d\epsilon}s \ , \end{aligned} \tag{9}$$

where $d\hat{\kappa}/d\epsilon$ may be expressed as a function of κ. To complete the constitutive assumptions, the yield function and $\hat{\kappa}(\epsilon)$ were chosen as follows.

†This scheme may be readily generalized to multidimensional states or to the case of dipolar stresses (WRIGHT & BATRA [1985]), since it turns out that all plastic rates may be related by a single proportional factor, as in GREEN & NAGHDI [1965] and GREEN, McINNIS & NAGHDI [1968]. Then it is the proportional factor that is determined from the analog of eqn (4) rather than $\dot{\gamma}_p$ itself.

$$|s| = \kappa(1 - a\theta)(1 + b\dot{\gamma}_p)^m ,$$
$$\hat{\kappa}(\epsilon) = \kappa_0\left(1 + \frac{\epsilon}{\epsilon_0}\right)^n , \tag{10}$$

where $\theta = T - T_0$. It will be recognized that the viscoplastic effect in the present case comes from a multiplicative overstress factor, although eqn (4) is sufficiently general to include an additive overstress or many other possibilities.

III. NONDIMENSIONAL VARIABLES AND HOMOGENEOUS SOLUTIONS

With nondimensional variables defined by

$$Y = H\bar{Y} , \quad u = H\bar{u} , \quad s = \kappa_0\bar{s} , \quad \theta = \frac{\kappa_0}{\rho c_v}\bar{\theta}$$
$$t = \frac{1}{\dot{\gamma}_0}\bar{t} , \quad \gamma = \bar{\gamma} , \quad \kappa = \kappa_0\bar{\kappa} , \quad \epsilon = \bar{\epsilon} , \tag{11}$$

where $\dot{\gamma}_0$ is the average strain rate imposed in the problem, the complete equations in nondimensional form become

$$\text{Momentum: } s,_Y = \frac{\rho H^2\dot{\gamma}_0^2}{\kappa_0}\ddot{u} ,$$

$$\text{Energy: } \dot{\theta} = \frac{k}{\rho c_v\dot{\gamma}_0 H^2}\theta,_{YY} + \kappa\dot{\epsilon} ,$$

$$\text{Constitutive: } \dot{s} = \frac{\mu}{\kappa_0}(\dot{\gamma} - \dot{\gamma}_p) ,$$
$$\kappa = \left(1 + \frac{\epsilon}{\epsilon_0}\right)^n , \tag{12}$$
$$\kappa\dot{\epsilon} = s\dot{\gamma}_p(=\dot{W}_p) ,$$

$$\text{Yield Surface: } |s| = \left(1 - \frac{a\kappa_0}{\rho c_v}\theta\right)(1 + b\dot{\gamma}_0\dot{\gamma}_p)^m\kappa ,$$

where the overbars have been dropped, and eqn $(12)_6$ is subject to eqn (5). There are two relative length scales implicit in eqns (12), namely a thermal length $(k/\rho c_v\dot{\gamma}_0H^2)^{1/2}$, and a viscous length $b/h(\kappa_0/\rho)^{1/2}$. In addition there are seven other nondimensional parameters in eqns (12) which are required to define the mass, elastic modulus, thermal softening, work hardening, and rate hardening of the material.

In a homogeneous deformation the true displacement field has the form $u = \dot{\gamma}_0 Yt$, where $\dot{\gamma}_0$ is a constant strain rate, or with nondimensional variables $u = Yt$, and nondimensional values of s, θ, γ_p, κ, and ϵ depend only on time. For this case the equations become ordinary differential equations with initial values

$$s(0) = 1 \ , \quad \theta(0) = 0 \ , \quad \epsilon(0) = 0 \ , \tag{13}$$

where time is counted from the first onset of plastic flow, and $\dot{\gamma}_p$ is to be found from eqn $(12)_6$. Equation $(12)_1$ is satisfied identically, and $(12)_2$ with $(12)_4$ substituted for κ can be integrated immediately to give $\theta(\epsilon)$. The remaining two equations may now be written as the autonomous pair

$$\begin{aligned} \dot{s} &= \frac{\mu}{\kappa_0}(1 - \dot{\gamma}_p) \ , \quad s(0) = 1 \ , \\ \dot{\epsilon} &= \frac{s\dot{\gamma}_p}{\left(1 + \frac{\epsilon}{\epsilon_0}\right)^n} \ , \quad \epsilon(0) = 0 \ , \end{aligned} \tag{14}$$

where

$$\begin{aligned} \theta(\epsilon) &= \frac{\epsilon_0}{1+n}\left[\left(1 + \frac{\epsilon}{\epsilon_0}\right)^{1+n} - 1\right] , \\ \dot{\gamma}_p &= \frac{1}{b\dot{\gamma}_0}\left[\left\{\frac{s}{\left(1 + \frac{\epsilon}{\epsilon_0}\right)^n\left(1 - \frac{a\kappa_0}{\rho c_v}\theta(\epsilon)\right)}\right\}^{1/m} - 1\right] . \end{aligned} \tag{15}$$

Although solutions to eqn (14) cannot be given explicitly, some of their features can be described qualitatively (see WRIGHT & BATRA [1985]). In particular, for the constitutive and yield functions chosen here, s always has a simple maximum at a critical value of γ, the exact value of which is influenced by work hardening, heat capacity, rate sensitivity, thermal softening, and yield strength, the first three tending to retard the peak, the last two to advance it. Figure 3 shows the homogeneous stress strain response for one particular choice of nondimensional parameters, as follows:

$$\frac{\rho\dot{\gamma}_0^2 H^2}{\kappa_0} = 3.928 \times 10^{-5} \ , \quad \frac{k}{\rho c_v H^2 \dot{\gamma}_0} = 3.978 \times 10^{-3} \ , \quad \frac{a\kappa_0}{\rho c_v} = 0.4973 \ ,$$

$$\frac{\mu}{\kappa_0} = 240.3 \ , \quad n = 0.09 \ , \quad \epsilon_0 = 0.017 \ , \quad \dot{\gamma}_0 b = 5 \times 10^5 \ , \quad m = 0.02 \ .$$

IV. RESPONSE TO PERTURBATIONS

Other analyses (e.g., BURNS [1983] or SHAWKI *et al.* [1983]) have indicated that if a small perturbation is added to the homogeneous response, its amplitude will begin to grow once the peak stress for homogeneous deformation has been passed. The perturbation could be applied to any of the field variables, but in this paper a small symmetric temperature bump was added at the center of the slab just before the peak stress, and

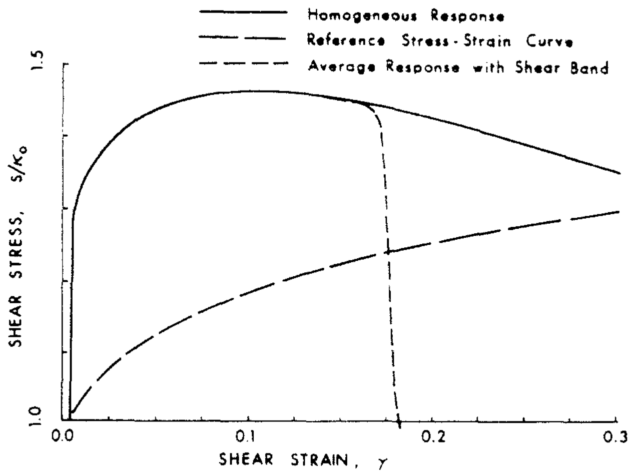

Fig. 3. Stress-strain response curves for a block of material in shear.

the problem was restarted as an initial/boundary value problem, the material parameters remaining exactly as before. The boundary values are

$$v(\pm 1, t) = \pm 1, \quad \theta,_Y(\pm 1, t) = 0 \ , \tag{16}$$

so that the average strain rate in the strip $[-1, +1]$ is maintained, and the strip is adiabatic overall. Equations $(12)_{1,2}$ with $\ddot{u}$ replaced by $\dot{v}$, govern the evolution of v and θ in the slab. In order to solve these equations subject to boundary conditions (16) and a suitable set of initial conditions by the finite element method, we first obtain a weak form of eqns $(12)_{1,2}$. Referring the reader to BECKER *et al.* [1981] and WRIGHT & BATRA [1985] for details, we merely mention that weak forms of eqns $(12)_{1,2}$ are

$$\begin{aligned} \hat{M}\dot{\tilde{v}} &= -\tilde{F} \ , \\ \hat{H}\dot{\tilde{\theta}} &= -\hat{T}\tilde{\theta} + \tilde{W} \end{aligned} \tag{17}$$

where superimposed hats and tildes signify square matrices and vectors or column matrices respectively. In order that the same finite element code could be used for both dipolar and nonpolar materials, we used C^1(Hermite) elements to approximate the velocity field and C^0 elements to interpolate the temperature field. Matrices $\tilde{M}$, $\tilde{F}$ etc. were evaluated by using the 4-point Gaussian quadrature rule. Equations (17), $(12)_3$ and $(12)_5$ with $\dot{\gamma}_p$ given by

$$\dot{\gamma}_p = \max\left\{\frac{1}{b\dot{\gamma}_0}\left(\left[\frac{|s|}{\left(1 - \dfrac{a\kappa_0}{\rho c_v}\theta\right)\kappa}\right]^{1/m} - 1\right), 0\right\}$$

were integrated with respect to time t by using the forward difference method and $\Delta t = .1 \times 10^{-6}$. Some of the principal results are shown in Figs. 4, 5, and 6.

Figure 4 shows a cross section of the temperature at various times after introduction of the perturbation. On this scale the peak in the initial temperature itself does not show since it is only 0.02 higher than the surrounding ambient value, which is reached at $Y = \pm 0.1$ on either side of the central peak. Only half of the central part is shown since the profile is symmetric and remains flat on out to ± 1. Cross sections of plastic strain rate also show a strong central peak at late times.

Figure 5 shows the stress, plastic strain rate, and temperature as functions of time at a point very near to the center of the band. The plastic rate begins a slow increase, which is actually exponential at first, and then after a fairly long run-in time, it accelerates rapidly, goes over an abrupt peak, and finally begins a slow decline. The temperature begins with a slow but steady increase and then rises very rapidly at the end, whereas the stress begins with a slow decrease and then drops rapidly at the end. It is during the period of most rapid change that the shear band takes recognizable shape.

Figure 6 shows the same three functions as in Fig. 5, but at a point near the bound-

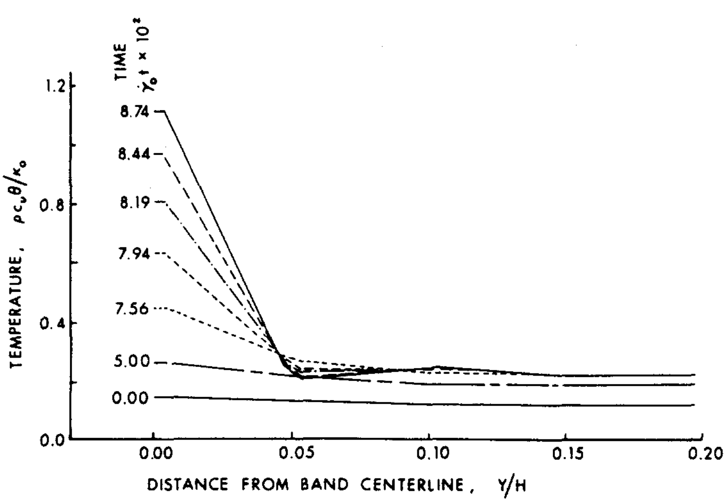

Fig. 4. Temperature distribution in a finite block of material at various times as a shear band forms.

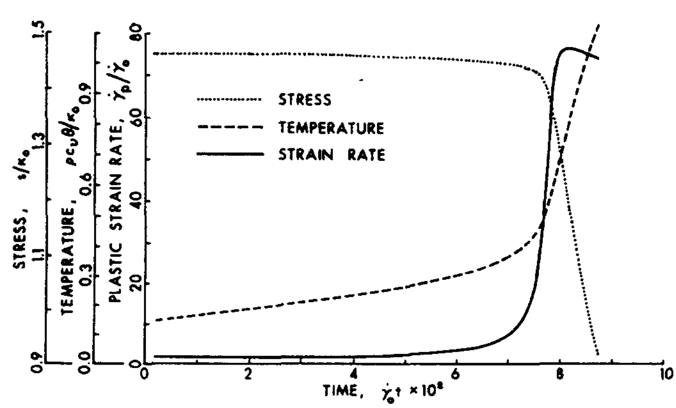

Fig. 5. Stress, temperature, and plastic strain rate vs. time near the center of a block of material as a shear band forms.

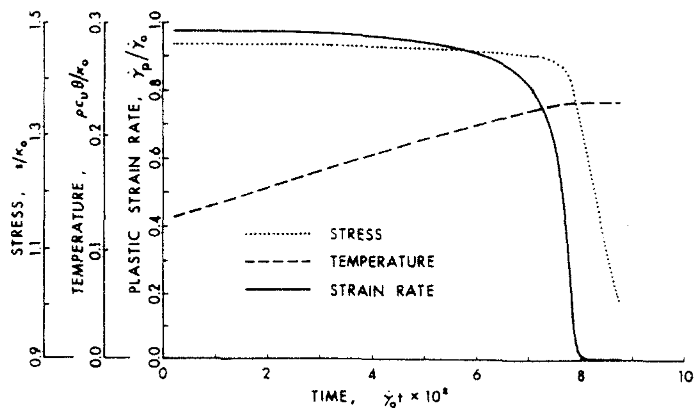

Fig. 6. Stress, temperature, and plastic strain rate vs. time near the edge of a block of material as a shear band forms.

ary. Here the plastic strain rate decreases slowly at first, and after the run-in time, it drops sharply and finally makes a rapid, but smooth transition to zero. Since plastic work ceases towards the end, the temperature arrives at a plateau, but the stress continues to drop on into the elastic region. Comparison of the stress curves shows that, although central and edge stresses are nearly equal during the run-in time, the edge stress actually drops later than the central stress. Thus the curves indicate that the stress in the center drops rapidly because of thermal softening, but the stress at the edge drops because of momentum transfer. Since the average strain rate is constant, the stress/time plot may be interpreted as a stress/average strain plot. This is shown in Fig. 3.

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